

Mathematics

6–12

Section 26

Mathematics 6–12

1 Knowledge of algebra

1. Identify graphs of linear inequalities on a number line.
2. Identify graphs of linear equations and inequalities in the coordinate plane.
3. Identify or interpret the slope and intercepts of a linear graph or a linear equation.
4. Determine the equation of a line, given the appropriate information such as two points, point-slope, slope-intercept, or its graph.
5. Solve problems involving the use of equations containing rational algebraic expressions.
6. Factor polynomials (e.g., the sum or difference of two cubes).
7. Rewrite radical and rational expressions into equivalent forms.
8. Perform the four basic operations on rational and radical expressions.
9. Solve equations containing radicals.
10. Multiply or divide binomials containing radicals.
11. Solve quadratic equations by factoring, graphing, completing the square, or using the quadratic formula, including complex solutions.
12. Solve problems using quadratic equations.
13. Use the discriminant to determine the nature of solutions of quadratic equations.
14. Determine a quadratic equation from known roots.
15. Identify the graphs of quadratic inequalities.
16. Solve real-world problems using direct and inverse variations.
17. Solve systems of linear equations or inequalities.
18. Formulate or identify systems of linear equations or inequalities to solve real-world problems.
19. Solve equations or inequalities involving absolute value.
20. Expand given binomials to a specified positive integral power.
21. Determine a specified term in the expansion of given binomials.

22. Solve polynomial equations by factoring.
23. Perform vector addition, subtraction, and scalar multiplication on the plane.
24. Solve real-world problems involving ratio or proportion.

2 Knowledge of functions

1. Interpret the language and notation of functions.
2. Determine which relations are functions, given mappings, sets of ordered pairs, rules, and graphs.
3. Identify the domain and range of a given function.
4. Identify the graph of special functions (i.e., absolute value, step, piecewise, identity, constant function).
5. Find specific values of a given function.
6. Estimate or find the zeros of a polynomial function.
7. Identify the sum, difference, product, and quotient of functions.
8. Determine the inverse of a given function.
9. Determine the composition of two functions.
10. Determine whether a function is symmetric, periodic, or even/odd.
11. Determine the graph of the image of a function under given transformations (i.e., translation, rotations through multiples of 90 degrees, dilations, and/or reflections over $y=x$ horizontal or vertical lines).

3 Knowledge of geometry from a synthetic perspective

1. Determine the change in the area or volume of a figure when its dimensions are altered.
2. Estimate measurements of familiar objects using metric or standard units.
3. Determine the relationships between points, lines, and planes, including their intersections.
4. Classify geometric figures (e.g., lines, planes, angles, polygons, solids) according to their properties.
5. Determine the measures of interior and exterior angles of any polygon.

6. Determine the sum of the measures of the interior angles and the sum of the measures of the exterior angles of convex polygons.
7. Identify applications of special properties of trapezoids, parallelograms, and kites.
8. Solve problems using the definition of congruent polygons and related theorems.
9. Solve problems using the definition of similar polygons and solids and related theorems.
10. Apply the Pythagorean theorem or its converse.
11. Use 30-60-90 or 45-45-90 triangle relationships to determine the lengths of the sides of triangles.
12. Calculate the perimeter, circumference, and/or area of two-dimensional figures (e.g., circles, sectors, polygons, composite figures).
13. Apply the theorems pertaining to the relationships of chords, secants, diameters, radii, and tangents with respect to circles and to each other.
14. Apply the theorems pertaining to the measures of inscribed angles and angles formed by chords, secants, and tangents.
15. Identify basic geometric constructions (e.g., bisecting angles or line segments, constructing parallels or perpendiculars).
16. Identify the converse, inverse, and contrapositive of a conditional statement.
17. Identify valid conclusions from given statements.
18. Classify examples of reasoning processes as inductive or deductive.
19. Determine the surface area and volume of prisms, pyramids, cylinders, cones, and spheres.
20. Identify solids and their related nets.

4 Knowledge of geometry from an algebraic perspective

1. Solve distance and midpoint problems involving two points, a point and a line, two lines, and two parallel lines.
2. Identify the directrix, foci, vertices, axes, and asymptotes of a conic section where appropriate.
3. Determine the center and the radius of a circle given its equation, and identify the graph.
4. Identify the equation of a conic section, given the appropriate information.

5. Use translations, rotations, dilations, or reflections on a coordinate plane to identify the images of geometric objects under such transformations.

5 Knowledge of trigonometry

1. Identify equations of graphs of circular/trigonometric functions and their inverses.
2. Solve problems involving circular/trigonometric function identities.
3. Interpret the graphs of trigonometric functions (e.g., amplitude, period, phase shift).
4. Solve real-world problems involving triangles using the law of sines or the law of cosines.
5. Use tangent, sine, and cosine ratios to solve right triangle problems.

6 Knowledge of statistics

1. Interpret graphical data involving measures of location (i.e., percentiles, stanines, quartiles).
2. Compute the mean, median, and mode of a set of data.
3. Determine whether the mean, the median, or the mode is the most appropriate measure of central tendency in a given situation.
4. Interpret the ranges, variances, and standard deviations for ungrouped data.
5. Interpret information from bar, line, picto-, and circle graphs; stem-and-leaf and scatter plots; and box-and-whisker graphs.
6. Interpret problems involving basic statistical concepts such as sampling, experimental design, correlation, and linear regression.

7 Knowledge of probability

1. Determine probabilities of dependent or independent events.
2. Predict odds of a given outcome.
3. Identify an appropriate sample space for an experiment.
4. Make predictions that are based on relative frequency of an event.
5. Determine probabilities using counting procedures, tables, tree diagrams, and formulas for permutations and combinations.

8 Knowledge of discrete mathematics

1. Find a specified term in an arithmetic sequence.
2. Find a specified term in a geometric sequence.
3. Determine the sum of terms in an arithmetic or geometric progression.
4. Solve problems involving permutations and combinations.
5. Evaluate matrix expressions involving sums, differences, and products.
6. Rewrite a matrix equation as an equivalent system of linear equations or vice versa.
7. Represent problem situations using discrete structures such as sequences, finite graphs, and matrices.

9 Knowledge of calculus

1. Solve problems using the limit theorems concerning sums, products, and quotients of functions.
2. Find the derivatives of algebraic, trigonometric, exponential, and logarithmic functions.
3. Find the derivative of the sum, product, quotient, or the composition of functions.
4. Identify and apply definitions of the derivative of a function.
5. Use the derivative to find the slope of a curve at a point.
6. Find the equation of a tangent line or a normal line at a point on a curve.
7. Determine if a function is increasing or decreasing by using the first derivative in a given interval.
8. Find relative and absolute maxima and minima.
9. Find intervals on a curve where the curve is concave up or concave down.
10. Identify points of inflection.
11. Solve problems using velocity and acceleration of a particle moving along a line.
12. Solve problems using instantaneous rates of change and related rates of change, such as growth and decay.
13. Find antiderivatives for algebraic, trigonometric, exponential, and logarithmic functions.

14. Solve distance, area, and volume problems using integration.
15. Evaluate an integral by use of the fundamental theorem of calculus.

10 Knowledge of number sense and mathematical structure

1. Apply the properties of real numbers: closure, commutative, associative, distributive, identities, and inverses.
2. Distinguish relationships between the complex number system and its subsystems.
3. Apply inverse operations to solve problems (e.g., roots vs. powers, exponents vs. logarithms).
4. Apply number theory concepts (e.g., primes, factors, multiples) in real-world and mathematical problem situations.
5. Identify numbers written in scientific notation, including the format used on scientific calculators and computers.

11 Knowledge of mathematics as communication

1. Identify statements that correctly communicate mathematical definitions or concepts.
2. Interpret written presentations of mathematics.
3. Select or interpret appropriate concrete examples, pictorial illustrations, and symbolic representations in developing mathematical concepts.

12 Knowledge of mathematics as reasoning

1. Identify reasonable conjectures.
2. Identify a counter example to a conjecture.
3. Identify simple valid arguments according to the laws of logic.
4. Identify proofs for mathematical assertions, including direct and indirect proofs, proofs by mathematical induction, and proofs on a coordinate plane.
5. Identify process skills: induction, deduction, questioning techniques, and observation-inference.

13 Knowledge of mathematical connections

1. Identify equivalent representations of the same concept or procedure (e.g., graphical, algebraic, verbal, numeric).
2. Interpret relationships between mathematical topics (e.g., multiplication as repeated addition, powers as repeated multiplication).
3. Interpret descriptions, diagrams, and representations of arithmetic operations.

14 Knowledge of instruction

1. Select appropriate resources for a classroom activity (e.g., manipulatives, mathematics models, technology, other teaching tools).
2. Identify methods and strategies for teaching problem-solving skills and applications (e.g., constructing tables from given data, guess-and-check, working backwards, reasonableness, estimation).

15 Knowledge of assessment

1. Identify students' errors, including multiple errors that result in correct or incorrect answers (e.g., algorithms, properties, drawings, procedures).
2. Identify appropriate alternative methods of assessment (e.g., performance, portfolios, projects).